



# Explorers!

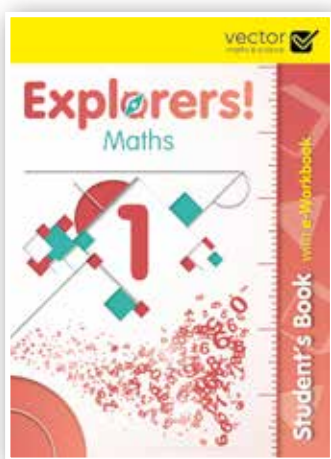
## Maths



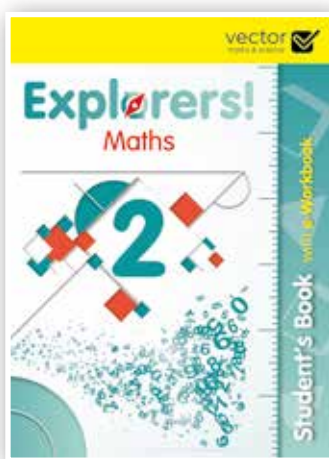
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# Explorers!

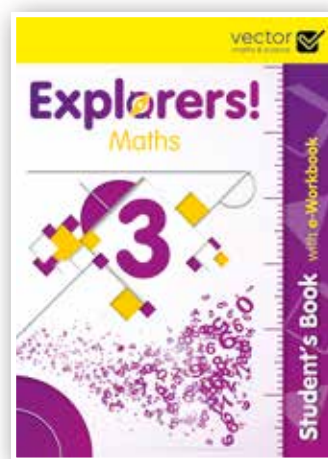
## Maths



LEVEL ONE



LEVEL TWO



LEVEL THREE

**Explorers! Maths** is a three-level lower secondary series, designed with the aim of ensuring that students attain a deep understanding of mathematical concepts and preparing them for higher education, by helping students respond positively towards all maths challenges. The series is based on the Singapore approach for maths and covers the learning objectives of various curriculums of Latin American countries.

**Explorers! Maths** series curriculum is structured into thematic units and satisfies five mathematical domains: numbers and algebra, geometry, measurement, statistics and probability and focuses on the progression of advanced mathematical skills. The series assists students in building mathematical knowledge and critical thinking through a wide variety of mathematical problems helping them to extend these skills in real-world contexts. This enhances students' confidence in their ability to tackle challenging problems and prepares them for their real-life future.

Levels 1-3

# Key features

## for STUDENTS

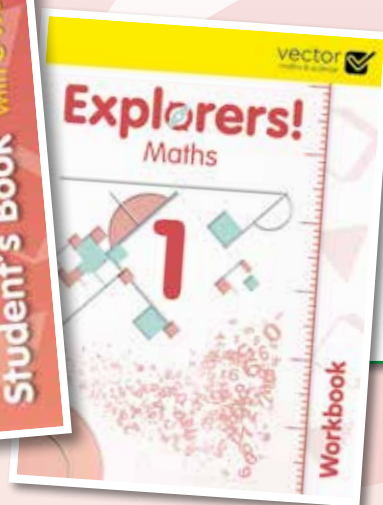
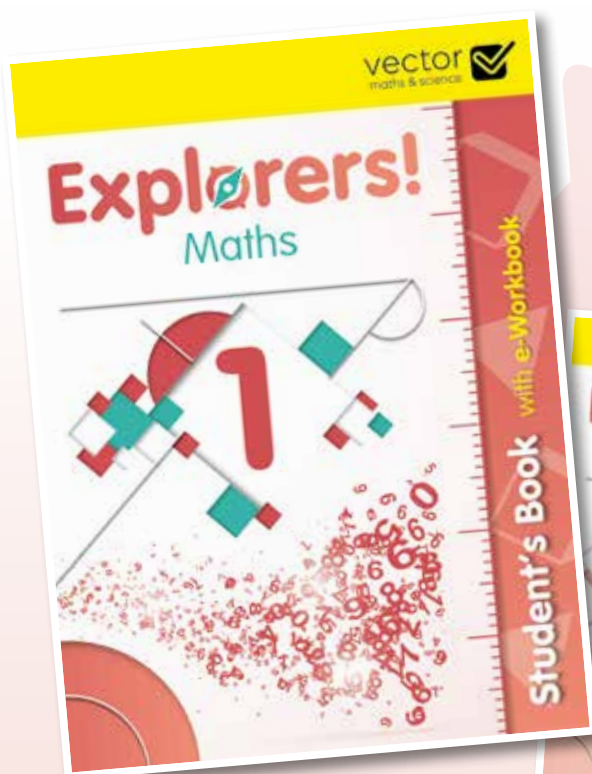
- A progressive development of mathematical knowledge and terminology
- Cover pages for each unit aimed at extending the mathematical concepts into real-life contexts
- Theory sections and worked examples to enable students to deeply understand the main mathematical concepts
- 'Apply your knowledge' sections to give students the opportunity to practise solving simple activities related to each unit
- 'Exercises' sections to facilitate expanding students' knowledge and testing their ability to solve more complicated activities
- 'More Exercises' sections to solve challenging activities which combine mathematical knowledge from previous units
- 'Note' sections to add information or specify the corresponding theory sections
- 'All about maths' sections to learn historical information related to the maths topics taught
- 'Using tech in maths...' sections to utilise technology to apply known mathematical methods
- 'Think deeper' sections allow students to challenge themselves to further explore main mathematical cores
- '!' sections to assist students in avoiding common mistakes
- 'Maths as language' sections to understand the meaning of the main mathematical symbols
- Assessment closed questions at the end of each unit
- Review pages in the middle and at the end of each level
- Glossary with age-appropriate definitions of critical mathematical terms at each level
- Downloadable and printable Workbook with activities for individual practice

## for TEACHERS

- Detailed maps of the Student's Book, Workbook and Teacher's Book that help the teacher understand the structure of each component
- A 'Map of the units' section that contains the theory sections, the learning objectives and the keywords to be covered in each unit organised in a table
- Step-by-step guidelines for each theory section and the supplementary sections ('Note', 'All about maths', 'Using tech in maths...', 'Think deeper' and '!') in the Student's Book
- The keys to all the activities of the 'Apply your knowledge', 'Exercises' and 'More Exercises' sections of the Student's Book
- The keys to all Assessment and Review pages

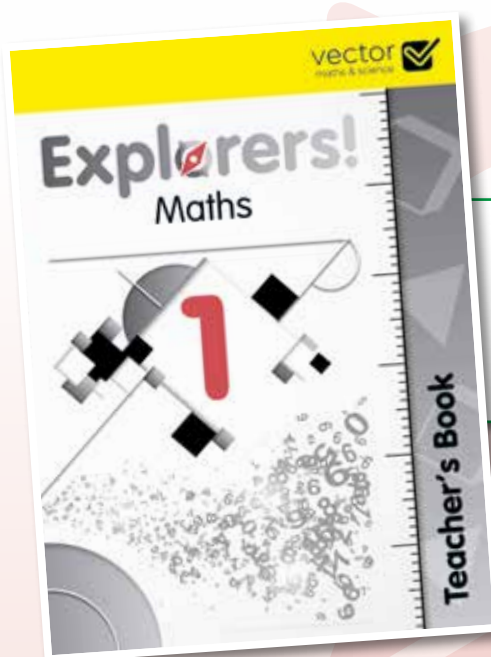
# Components

## > for Students



Student's  
Digital  
Resources

## > for Teachers



Teacher's  
Digital  
Resources

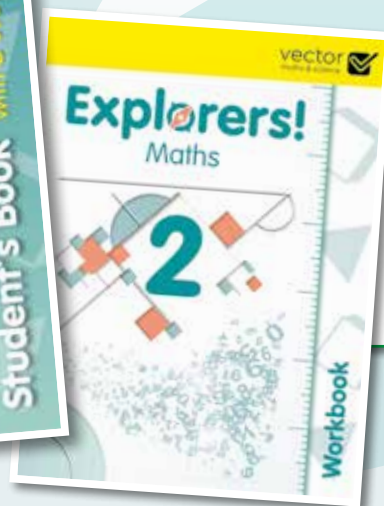
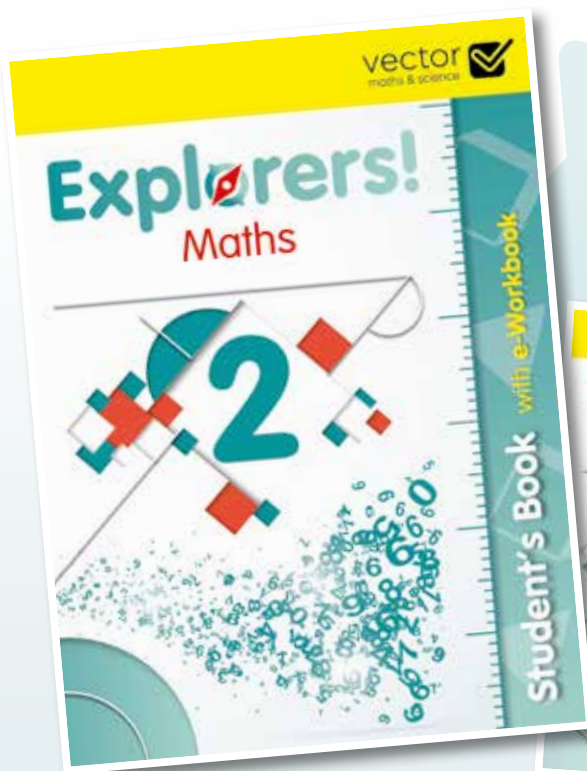


Theory sections	Learning objectives
1.1 The Number system 1.2 The Number line 1.3 Single operations with integers 1.4 Absolute value 1.5 Combined operations of integers 1.6 Laws of the four operations	<ul style="list-style-type: none"> <li>• Recognise natural numbers and integers.</li> <li>• Recognise the place value of each digit in integers.</li> <li>• Determine the position and order of integers on a number line.</li> <li>• Perform addition, subtraction, multiplication and division with integers.</li> <li>• Understand the properties and the order of the four operations.</li> <li>• Realise the different readings that addition and subtraction may have according to the context.</li> <li>• Solve word problems involving integers in different contexts.</li> <li>• Apply logical reasoning and critical thinking to mathematical concepts.</li> </ul>
2.1 Fractions 2.2 Comparing fractions 2.3 Mixed numbers and improper fractions 2.4 Multiplication and division of fractions and mixed numbers 2.5 Fraction of a quantity 2.6 Division of fractions and mixed numbers 2.7 Expressing one quantity as a fraction of another 2.8 Percentages 2.9 Word problems with percentages	<ul style="list-style-type: none"> <li>• Recall what a fraction is and recognise the numerator and denominator.</li> <li>• Recall proper fractions, improper fractions and mixed numbers and conversions between them.</li> <li>• Do calculations with fractions and mixed numbers.</li> <li>• Use pictorial representations of percentages.</li> <li>• Recognise the link between fractions, decimals and percentages.</li> <li>• Write fractions and decimals as percentages and vice versa.</li> <li>• Recognise percentages greater than 100%.</li> <li>• Solve problems involving multiplication and division of fractions and positive decimals in different contexts.</li> <li>• Use technological tools and pictorial representations for problem solving.</li> <li>• Solve simple word problems involving percentages.</li> </ul>
3.1 Decimals 3.2 Decimals and fractions 3.3 Recurring decimals 3.4 Ascending and descending order 3.5 Multiplication of decimals 3.6 Division of decimals 3.7 Problem solving with decimals	<ul style="list-style-type: none"> <li>• Recognise decimal numbers.</li> <li>• Recognise the place value of each digit in decimals.</li> <li>• Compare and put decimals in ascending or descending order.</li> <li>• Perform multiplication and division with decimals.</li> <li>• Write fractions as decimals and vice versa.</li> <li>• Solve word problems involving multiplication and division of decimals in different contexts.</li> </ul>
4.1 Introduction to exponents 4.2 Decomposition using powers of 10 4.3 Scientific notation	<ul style="list-style-type: none"> <li>• Familiarise ourselves with terms or expressions such as exponent, base, number raised to the power of ..., etc.</li> <li>• Use numbers written in specific notation that have base number 10 and are raised to natural exponents.</li> <li>• Familiarise ourselves with positive, negative and zero exponents.</li> <li>• Realise that any base, different from zero, raised to 0 equals 1.</li> <li>• Analyse numbers using powers of 10.</li> <li>• Recognise scientific notation as an easy way to represent extremely big or small numbers.</li> <li>• Solve problems using scientific notation.</li> </ul>
5.1 Algebraic expressions 5.2 Properties 5.3 Factorisation using the distributive property	<ul style="list-style-type: none"> <li>• Realise that we can use letters to represent numbers or variables.</li> <li>• Use given values for variables to evaluate algebraic expressions.</li> <li>• Differentiate between like and unlike terms and realise what kind of calculations we can perform between them.</li> <li>• Use commutative, associative, distributive and identity properties to simplify linear or more complex algebraic expressions with natural coefficients.</li> <li>• Realise that properties show us the correct way to do calculations and can be applied in both directions.</li> <li>• Realise that the distributive property is useful for factorising and expanding linear expressions such as <math>c(a + b)</math>, <math>ka + kb</math>, <math>ax + bx + kya + kyb</math>, etc.</li> <li>• Use algebraic expressions to represent real-life problems, phrases or situations constructing suitable equations.</li> </ul>

Theory sections	Learning objectives
<p>6.1 The Cartesian coordinate system</p> <p>6.2 Translations using vectors</p> <p>6.3 Graphs</p> <p>6.4 Linear graphs</p> <p>6.5 Applications of graphs of linear equations</p>	<ul style="list-style-type: none"> <li>Familiarise ourselves with the Cartesian coordinate system in two dimensions.</li> <li>Use ordered pairs to identify the position of a point on the coordinate plane.</li> <li>Use vectors to translate points or 2D shapes on the coordinate plane.</li> <li>Recognise direct and inverse proportion by making value tables or plotting ordered pairs on the Cartesian plane.</li> <li>Perceive graphs as the representation of the relation between two variables.</li> <li>Distinguish between graphs that represent direct and inverse proportion and use their features to answer questions.</li> <li>Apply real-life and mathematical problems to linear graph equations.</li> </ul>
<p>7.1 Definition of linear equations</p> <p>7.2 How to solve a linear equation</p> <p>7.3 Evaluation of formulas</p> <p>7.4 Problem solving with algebra</p> <p>7.5 Linear inequalities</p> <p>7.6 Solving linear inequalities with one unknown</p> <p>7.7 Problem solving involving inequalities</p>	<ul style="list-style-type: none"> <li>Identify an equation as a mathematical statement that has two mathematical expressions separated by the equals sign.</li> <li>Identify an inequality as a mathematical expression with two parts separated by the inequality sign.</li> <li>Distinguish between linear and non-linear expressions.</li> <li>Solve simple linear equations and inequalities.</li> <li>Use and evaluate formulas and other mathematical expressions.</li> <li>Familiarise ourselves with useful properties of linear inequalities.</li> <li>Model and solve real-life word problems by forming linear equations and inequalities such as <math>ax = b</math>, <math>\frac{x}{a} = b</math>, <math>ax &lt; b</math>, <math>ax &gt; b</math>, <math>\frac{x}{a} &lt; b</math>, <math>\frac{x}{a} &gt; b</math>, <math>a \neq 0</math>, etc.</li> </ul>
<p>8.1 Parallel and perpendicular lines</p> <p>8.2 Angle bisector</p> <p>8.3 Triangles</p> <p>8.4 Quadrilaterals</p> <p>8.5 Polygons</p> <p>8.6 Circles</p> <p>8.7 Special segments in triangles</p> <p>8.8 Perimeter and area of polygons</p> <p>8.9 Circumference and area of circles</p> <p>8.10 Perimeter and area problems</p>	<ul style="list-style-type: none"> <li>Recognise parallel and perpendicular lines.</li> <li>Identify and classify triangles according to their properties (e.g. side length, angle size, etc.).</li> <li>Recognise elements such as the bisector of an angle and the medians and altitudes of a triangle.</li> <li>Identify and classify quadrilaterals according to their properties (parallel sides, equal sides, etc.).</li> <li>Use formulas to calculate the area of quadrilaterals (parallelograms, trapeziums, etc.).</li> <li>Describe the relationship between the interior and the exterior angles of different types of polygons (triangles, quadrilaterals, pentagons, etc.).</li> <li>Identify basic elements of circles and the relationships between them (centre, radius, diameter, circumference, etc.).</li> <li>Recognise the shape of a circle as a set of points on a plane that share specific properties.</li> <li>Familiarise ourselves with special circles (e.g. circumcircle, incircle) and points (e.g. circumcentre, incentre) on the plane.</li> <li>Use knowledge of 2D shapes to solve word and real-life problems that include composite shapes.</li> <li>Familiarise ourselves with the number <math>\pi</math> as the ratio circumference : diameter of any circle.</li> <li>Use an approximation of <math>\pi</math> to calculate the circumference and the area of a circle.</li> <li>Use digital tools to construct 2D shapes and investigate their properties (e.g. finding the orthocentre and the incentre of a triangle, sketching the altitudes of a triangle, etc.).</li> </ul>

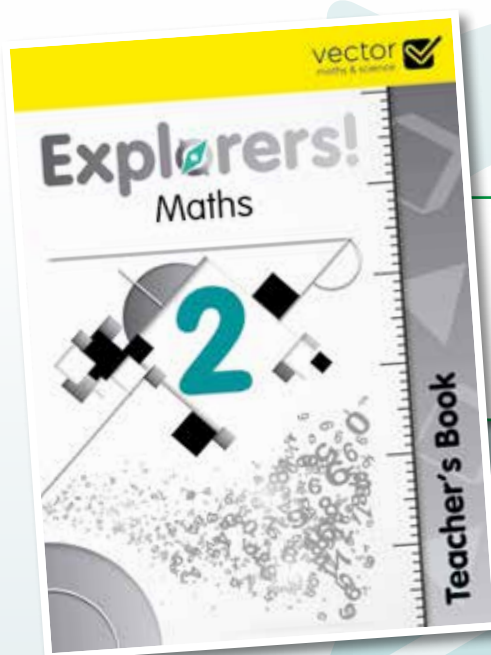
Theory sections	Learning objectives
9.1 Congruent figures 9.2 Congruent triangles 9.3 Tests for congruent triangles 9.4 Bisecting a line segment 9.5 Constructing triangles 9.6 Quadrilateral constructions	<ul style="list-style-type: none"> <li>• Recognise congruent figures.</li> <li>• Realise that congruent figures may have different orientations but be identical in shape.</li> <li>• Use tests of congruency to verify that two triangles are congruent.</li> <li>• Practise working with congruent triangles and quadrilaterals.</li> <li>• Practise making geometrical constructions with the use of compasses, a ruler and a protractor (draw an arc, a circle, bisect a line segment or an angle, draw a line perpendicular to a straight line, etc.).</li> <li>• Solve problems involving congruent triangles and other congruent figures.</li> </ul>
10.1 Basics of statistics 10.2 Tallies and frequency tables 10.3 Relative and cumulative frequencies 10.4 Pictograms 10.5 Bar charts 10.6 Pie charts 10.7 Line graphs	<ul style="list-style-type: none"> <li>• Recognise the influence of statistics in our daily life.</li> <li>• Collect, classify and organise data in tables.</li> <li>• Use tallies and absolute and cumulative frequency in tables, and represent data in suitable charts or graphs.</li> <li>• Construct, read and interpret pictograms, pie charts, bar charts and line graphs.</li> <li>• Recognise the advantages and disadvantages of the different ways of representing data when using statistics.</li> <li>• Comprehend how using the wrong graphs and tables or using them in the wrong way can lead us to interpret the data incorrectly.</li> </ul>
11.1 Data analysis 11.2 The mean of ungrouped data 11.3 Median 11.4 Mode 11.5 The mean of grouped data	<ul style="list-style-type: none"> <li>• Identify the mean, the mode and the median of a set of data.</li> <li>• Realise the purposes and uses of the mean, the mode and the median of a set of data.</li> <li>• Calculate all the different types of average and gain information about a data set.</li> <li>• Realise that when the mean of a data set is not typical of the data values, it is useful to find the median to extract safe information for the data set.</li> <li>• Draw conclusions about the population of a survey through different types of averages.</li> </ul>
12.1 Introduction to probability 12.2 Definitions in probability 12.3 Properties of probability 12.4 Tree diagrams	<ul style="list-style-type: none"> <li>• Familiarise ourselves with the concept of probability as a measure of chance.</li> <li>• Familiarise ourselves with the random experiment, the outcome, the sample space and the event.</li> <li>• Use a formula to find the probability of an event.</li> <li>• Use the properties of probability to solve problems.</li> <li>• Calculate the probability of a single event.</li> <li>• Compare the probability of an event at random with its relative frequency in a digital experiment.</li> <li>• Use tables, tree diagrams, etc. to represent all possible outcomes of an experiment.</li> </ul>

## > for Students



Student's  
Digital  
Resources

## > for Teachers



Teacher's  
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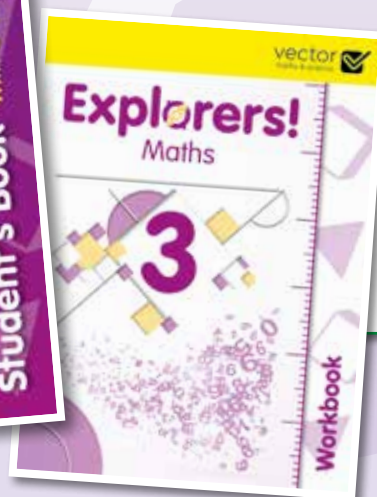
Theory sections	Learning objectives
1.1 Integers	<ul style="list-style-type: none"> <li>• Use visual representations (moving on the number line, grouping units to divide, etc.) to perform the four operations applying the sign rules for integers.</li> <li>• Recognise terminating decimals and distinguish them from decimals with an infinite number of non-repeating decimal places.</li> <li>• Apply the algorithm of division to convert fractions to decimals.</li> <li>• Identify rational numbers as the numbers that can be written in the form of a fraction.</li> <li>• Realise that recurring decimals are rational numbers.</li> <li>• Practise operating with rational numbers and represent their operations on the number line.</li> <li>• Perform a combination of operations between all known types of numbers (integers, fractions, decimals).</li> <li>• Solve real-life and mathematical problems involving rational numbers and percentages included.</li> </ul>
1.2 Single operations with integers	
1.3 Rational numbers	
1.4 Decimals, fractions and rational numbers	
1.5 Single operations with rational numbers	
1.6 Combined operations with rational numbers	
1.7 Percentages	
2.1 Introduction to exponents	<ul style="list-style-type: none"> <li>• Identify numbers written in exponent form with base and exponents being natural numbers.</li> <li>• Familiarise ourselves with the multiplication and division laws for exponents.</li> <li>• Use pictorial representations to comprehend how numbers in exponent form are generated.</li> <li>• Identify the existence of square roots for natural numbers.</li> <li>• Represent an approximation of non-natural square roots on the number line.</li> <li>• Use pictorial and geometrical representations in order to give meaning to irrational square roots.</li> <li>• Solve word, real-life and geometrical problems involving exponents and square roots.</li> </ul>
2.2 Laws of exponents	
2.3 Square roots	
3.1 Algebraic expressions	<ul style="list-style-type: none"> <li>• Practise performing proper operations between various types of terms applying basic properties of calculations.</li> <li>• Familiarise ourselves with the concepts of factorisation and expansion of algebraic expressions.</li> <li>• Realise that the distributive property is useful for factorising and expanding linear expressions such as <math>c(a + b)</math>, <math>ka + kb</math>, <math>ax + bx + kya + kyb</math>, etc.</li> <li>• Practise performing factorisation and expansion using different methods (common factors, grouping, special products, etc.).</li> <li>• Use pictorial and geometrical representations in order to give meaning to algebraic expressions.</li> <li>• Use algebraic expressions to solve geometrical problems involving areas and volumes.</li> </ul>
3.2 Expansion of algebraic expressions	
3.3 Factorisation of algebraic expressions	
3.4 Problem solving involving algebraic expressions	
4.1 Linear equations	<ul style="list-style-type: none"> <li>• Practise solving linear equations and inequalities, with rational coefficients and constants, applying the corresponding properties.</li> <li>• Solve linear equations and inequalities with rational coefficients and constants.</li> <li>• Model and solve mathematical or real-life word problems forming linear equations such as <math>ax = b</math>, <math>\frac{x}{a} = b</math>, <math>\frac{a}{x} + b = c</math>, <math>a(x + b) = c</math>, <math>ax + b = cx + d</math>, etc. where <math>a \neq 0</math> and <math>a, b, c, d</math> are rational numbers.</li> <li>• Model and solve mathematical or real-life word problems forming linear inequalities and represent their solutions on the number line.</li> <li>• Use technological tools when solving equations and inequalities.</li> </ul>
4.2 Linear equations with decimals	
4.3 Problem solving involving linear equations	
4.4 Linear inequalities	
4.5 Problem solving involving inequalities	

Theory sections	Learning objectives
5.1 Introduction to functions 5.2 Linear functions and gradient 5.3 Affine function 5.4 Applications of graphs of functions	<ul style="list-style-type: none"> <li>• Familiarise ourselves with the notion of a function as a special type of relation between two variables.</li> <li>• Use arrow diagrams, value tables, etc. to represent the functions that link two variables.</li> <li>• Realise that a function is like a machine that processes the input information and provides the corresponding output.</li> <li>• Recognise the general form of linear and affine functions and study their key features.</li> <li>• Draw a graph of a linear or affine function on a coordinate plane.</li> <li>• Realise that the graph of an affine function is the vertical translation of a parallel linear function graph.</li> <li>• Realise that we can produce an affine function by adding a constant to a linear function.</li> <li>• Understand the gradient of a slope as the ratio of vertical change of the <math>y</math>-coordinates to horizontal change of the <math>x</math>-coordinates.</li> <li>• Use technological tools and pictorial representations for problem solving.</li> <li>• Apply real-life or mathematical problems to linear or affine functions.</li> </ul>
6.1 3D shapes 6.2 Nets 6.3 Volume and total surface area of common solids	<ul style="list-style-type: none"> <li>• Recognise the solids for a given net and imagine the nets of various solids.</li> <li>• Calculate the total surface area and volume of various prisms, cylinders, etc.</li> <li>• Realise that all right prisms have the same formula for volume which is area of base by height.</li> <li>• Apply total surface area and volume of 3D shapes formulas to real-life or mathematical problems.</li> </ul>
7.1 Pythagoras' theorem 7.2 Determine if a triangle is right-angled 7.3 Applications of Pythagoras' theorem	<ul style="list-style-type: none"> <li>• Realise that the sentence of a theorem states a truth that is a mathematical statement.</li> <li>• Perceive Pythagoras' theorem as a mathematical statement.</li> <li>• Familiarise ourselves with the geometrical interpretation of Pythagoras' theorem.</li> <li>• Use the algebraic expression of Pythagoras' theorem to calculate the unknown length of a side of a right-angled triangle.</li> <li>• Perceive the converse of Pythagoras' theorem as the mathematical foundation to determine whether a triangle is right-angled or not.</li> <li>• Use technological tools and pictorial representations for deeper comprehension of Pythagoras' theorem and problem solving.</li> <li>• Solve mathematical and real-life problems using Pythagoras' theorem.</li> </ul>
8.1 Translation 8.2 Reflection 8.3 Rotation 8.4 Transformations of 3D shapes	<ul style="list-style-type: none"> <li>• Familiarise ourselves with translation, reflection and rotation and their properties.</li> <li>• Use vectors to translate points or 2D shapes on the coordinate plane.</li> <li>• Use lines that are horizontal, vertical or at any other angle as axes of reflection on the coordinate plane to reflect 2D shapes.</li> <li>• Perform rotation of a shape about a centre of rotation.</li> <li>• Use ordered pairs to identify and describe the position and the transformation of 2D shapes on the coordinate plane.</li> <li>• Perform, one after another, different translations of a 2D shape on the coordinate plane.</li> <li>• Recognise the planes of symmetry of a 3D shape.</li> <li>• Perform translation and rotation on 2D shapes to produce 3D shapes.</li> <li>• Use technological tools to perform and represent translations on the coordinate plane.</li> </ul>

Theory sections	Learning objectives
9.1 Dot diagrams 9.2 Histograms 9.3 Stem-and-leaf diagrams 9.4 Pie charts 9.5 Measures of position and box plot	<ul style="list-style-type: none"> <li>• Construct, read and interpret dot diagrams, histograms, stem-and-leaf diagrams, pie charts and box plots.</li> <li>• Identify the quartiles and percentiles as measures of position.</li> <li>• Divide ascending order data sets into quartiles and percentiles and recognise what parts of the data are smaller than, between, or greater than them.</li> <li>• Represent sets of data using box plots and use their depiction as an easy way to compare the spread of a data set between the maximum and minimum value.</li> <li>• Realise the advantages and disadvantages of the different ways of representing data when using statistics.</li> <li>• Comprehend how using the wrong graphs, charts and tables or using them in the wrong way can lead us to interpret the data incorrectly.</li> </ul>
10.1 Multiplication principle 10.2 Measuring probability	<ul style="list-style-type: none"> <li>• Draw tree diagrams and form tables to represent the sample space of an experiment.</li> <li>• Familiarise ourselves with experiments consisting of multiple successive stages.</li> <li>• Use a formula to find the probability of a multiple step event.</li> <li>• Recognise the multiplication principle as a way to find the number of all possible outcomes of an experiment consisting of successive stages.</li> </ul>

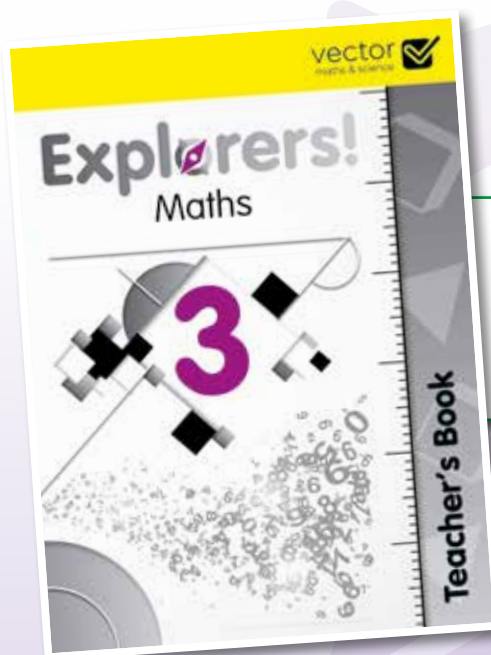
# Components

## > for Students



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## > for Teachers



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Resources



Theory sections	Learning objectives
1.1 Rational numbers 1.2 Addition and subtraction with rational numbers 1.3 Multiplication and division with rational numbers 1.4 Combined operations with rational numbers	<ul style="list-style-type: none"> <li>Identify different sets of numbers and the symbols we use to denote them.</li> <li>Identify rational numbers as the numbers that can be written in the form of a fraction with integers in both the numerator and denominator.</li> <li>Classify numbers into different sets according to their properties.</li> <li>Realise the relation between different sets such as the set of rational numbers includes integers, the set of integers includes natural numbers, etc.</li> <li>Apply commutative, associative and distributive properties to make calculations and simplify arithmetic expressions involving rational numbers.</li> <li>Perform combined operations involving rational numbers.</li> <li>Convert mathematical expressions to natural language and vice versa.</li> <li>Solve real-life and mathematical problems involving rational numbers.</li> </ul>
2.1 Powers with integer exponents 2.2 Laws of powers 2.3 Solving real-life problems involving powers	<ul style="list-style-type: none"> <li>Familiarise ourselves with negative integer or zero exponents.</li> <li>Realise that we can extend the laws of powers to numbers in exponent form with base any rational number.</li> <li>Perform combined operations involving the laws of exponents.</li> <li>Identify exponential growth and decay and distinguish them from linear increase and decrease.</li> <li>Use visual representations of the functions of exponential growth and decay for their further understanding.</li> <li>Model and solve real-life word problems involving exponential growth and decay.</li> </ul>
3.1 Monomials and polynomials 3.2 Special products 3.3 Factorisation using special products 3.4 Solving problems involving special products	<ul style="list-style-type: none"> <li>Identify monomials as terms of algebraic expressions.</li> <li>Realise that monomials with different variable parts are unlike terms that cannot be added or subtracted.</li> <li>Identify polynomials and familiarise ourselves with special vocabulary of polynomials with one, two or three unlike terms.</li> <li>Perform operations with polynomials properly, applying basic properties of calculations (e.g., commutative, associative, and distributive properties).</li> <li>Practise performing factorisation and expansion of algebraic expressions using special products.</li> <li>Recognise and perform operations with special products that involve complex binomials.</li> <li>Use geometrical representations to visualise and verify the equalities of the special products composing or splitting the area of rectangles and squares.</li> <li>Use the geometrical representations of special products to convert algebraic expressions from the form of a product to a sum and vice versa.</li> <li>Solve word or real-life geometrical problems combining previous knowledge in geometry and special products.</li> </ul>
4.1 Linear equations in the form $ax + by = k$ 4.2 Graphical solution to simultaneous linear equations 4.3 Algebraic solution to simultaneous linear equations 4.4 Solving word problems involving simultaneous equations 4.5 Linear relations in the form $f(x, y) = ax + by$	<ul style="list-style-type: none"> <li>Identify a linear equality in the form <math>ax + by = k</math> where <math>a, b, k \in \mathbb{Q}</math>, as a linear function and make value tables to represent them on the coordinate plane or answer questions.</li> <li>Explore the position change of a linear equality in the form <math>ax + by = k</math> given <math>a, b, k \in \mathbb{Q}</math> for a variety of values for <math>k</math>.</li> <li>Recognise <math>y = mx + c</math> and <math>ax + by = k</math> as equivalent general forms for linear equations.</li> <li>Solve simultaneous linear equations with two variables, using the elimination, substitution, equality or graphical method.</li> <li>Familiarise ourselves with systems of simultaneous linear equalities with no, one, or an infinite number of solutions and relate them to different relative positions of lines on the coordinate plane.</li> <li>Recognise <math>f(x, y) = ax + by</math> as a linear relation and realise that it can be visualised as a straight line on the coordinate plane, when given the value of <math>f(x, y)</math>.</li> <li>Solve word and real-life problems that involve simultaneous linear equations using algebraic methods or/and geometrical representations on the coordinate plane.</li> <li>Use educational tools to solve problems, verify solutions or generally experiment on simultaneous linear equations systems.</li> <li>Model various real-life situations involving simultaneous linear equations systems.</li> </ul>

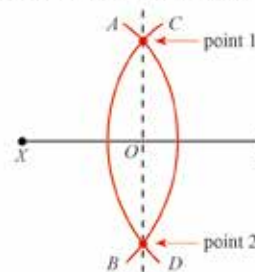
Theory sections	Learning objectives
5.1 Features and construction of a circle 5.2 Arc length and sector area 5.3 Circular segments and perimeters 5.4 Cones	<ul style="list-style-type: none"> <li>• Identify the central angle, the circular sector and the circular segment of a circle and realise the relationship between them.</li> <li>• Familiarise ourselves with step by step guidelines to reach a formula for calculating a central angle of a circular sector equal to <math>60^\circ</math>, <math>90^\circ</math>, <math>120^\circ</math>, etc.</li> <li>• Familiarise ourselves with the geometric construction of equal consecutive circular sectors and central angles.</li> <li>• Calculate the length of an arc, the perimeter of a circular sector or a circular segment and find the length of more complicated regions including circular sectors and segments.</li> <li>• Calculate the area of a circular sector or a circular segment and find the area of more complicated regions including circular sectors and segments.</li> <li>• Recognise a cone as a solid and use the appropriate terminology for its features.</li> <li>• Calculate the total surface area and volume of cones with different radii and heights using formulas.</li> <li>• Solve word, real-life and geometric problems involving arcs, circular sectors and segments as well as cones.</li> </ul>
6.1 Proportional line segments 6.2 Thales's theorem 6.3 Homothety 6.4 Vectors and homothety 6.5 Applications	<ul style="list-style-type: none"> <li>• Familiarise ourselves with proportional line segments and link this concept with vectors and scalar multiples.</li> <li>• Perceive Thales's theorem as a mathematical statement.</li> <li>• Familiarise ourselves with the geometrical interpretation of Thales's theorem.</li> <li>• Use Thales's theorem for triangles and its reciprocal to solve triangles.</li> <li>• Make geometrical constructions to divide line segments into <math>n</math> equal parts with a given ratio applying Thales's theorem.</li> <li>• Identify the concept of homothety and familiarise ourselves with the ratio of homothety.</li> <li>• Use a given ratio to draw the homothetic image of a figure.</li> <li>• Use scalar multiples to draw the homothetic image of a figure.</li> <li>• Use technological tools and pictorial representations for deeper comprehension of homothety and Thales's theorem.</li> <li>• Solve mathematical and real-life problems using homothety, Thales's theorem and their properties.</li> </ul>
7.1 Similar figures 7.2 Similar polygons 7.3 Similar triangles 7.4 Euclid's theorem 7.5 Solving problems with similar triangles	<ul style="list-style-type: none"> <li>• Familiarise ourselves with the concept of similarity between figures.</li> <li>• Realise that similar figures have the same shape and can have the same size or can be an enlargement or reduction of each other.</li> <li>• Use the conditions of similarity to identify similar polygons and calculate unknown sides or angles.</li> <li>• Realise that there are three criteria to verify that two triangles are similar.</li> <li>• Perceive Euclid's theorem as a mathematical statement.</li> <li>• Familiarise ourselves with the geometrical interpretation of Euclid's theorem.</li> <li>• Apply Euclid's theorem to similar triangles and realise that the corollary of Euclid's theorem is a tool to identify parallel line segments in shapes with triangles.</li> <li>• Solve mathematical and real-life problems using similarity, Euclid's theorem, and their properties.</li> </ul>

Theory sections	Learning objectives
8.1 Scatter plots with quantitative variables 8.2 Comparison of populations 8.3 Contingency table	<ul style="list-style-type: none"> <li>• Realise how quantitative and qualitative variables differ through different methods of analysing and interpreting data.</li> <li>• Depict two characteristics of a population using points on the coordinate plane.</li> <li>• Realise that the cloud of dots is a diagram used to examine the relationship between two variables.</li> <li>• Draw and analyse scatter plots and make conclusions about the relationship between characteristics of populations.</li> <li>• Use two colours to represent dots of different populations on a scatter plot.</li> <li>• Use scatter plots to compare populations by different characteristics and calculate different types of average to gain further information.</li> <li>• Intuitively draw a line to highlight the tendency of a cloud of dots or to separate the dots of different populations on a scatter plot.</li> <li>• Familiarise ourselves with contingency tables and realise their differences from frequency tables.</li> <li>• Use contingency tables to conclude the relationship of at least two qualitative variables.</li> <li>• Make calculations of probabilities with data of the contingency table to realise what the combination of frequencies is.</li> <li>• Solve real-life surveys word problems using scatter plots and contingency tables.</li> </ul>
9.1 Probability and properties 9.2 Addition rules for probability 9.3 Multiplication rules for probability 9.4 Tree diagrams	<ul style="list-style-type: none"> <li>• Identify Venn diagrams as a way to represent the relationship between all possible outcomes of a random experiment.</li> <li>• Familiarise ourselves with the concepts of union and intersection of events with the use of Venn diagrams.</li> <li>• Familiarise ourselves with the multiplication and addition rules of probabilities.</li> <li>• Use Venn diagrams to calculate the probability of an event using the multiplication and addition rules or a combination of them.</li> <li>• Recognise mutually and non-mutually exclusive events and calculate the probability of their union.</li> <li>• Recognise conditional probability and use the formula to calculate its measure.</li> <li>• Realise that independent events do not affect the probability of other events happening.</li> <li>• Draw tree diagrams to represent the total possible outcomes of a random experiment with successive events.</li> <li>• Solve real-life word problems involving probabilities.</li> </ul>
10.1 Random walks 10.2 Probability in random walks	<ul style="list-style-type: none"> <li>• Familiarise ourselves with the concept of random walks as a process where an object moves through a series of random steps in a given space.</li> <li>• Realise that in a random walk, the probabilities of the outcomes happening are equal at each stage.</li> <li>• Use tree diagrams to visualise all possible outcomes of a random experiment.</li> <li>• Use probabilities in general as well as addition and multiplication rules in random walk experiments.</li> <li>• Identify the Galton board as a physical device that simulates the distribution in random walk experiments.</li> <li>• Realise the connection between random walks and the use of a Galton board.</li> <li>• Identify empirical probability and realise that is equivalent to the relative frequency of an observation.</li> <li>• Use technological tools to further study random walks and Galton board application.</li> <li>• Solve real-life problems involving random walks.</li> </ul>

## 9 Congruent figures and constructions

### 9.4 Bisecting a line segment

To bisect a line segment using a pair of compasses and a ruler, we draw a perpendicular line that crosses through the middle point of the line segment.



- Step 1:** Place the compass at one end of the line segment, point X.  
**Step 2:** Adjust the compass to slightly longer than half the line segment.  
**Step 3:** Draw arc AB.  
**Step 4:** Keeping the compass at the same length, draw arc CD with the compass at point Y.  
**Step 5:** Join point 1 to point 2 cutting the line segment XY at O.  
**Step 6:** The line segments XO and OY have the same length. So, we bisected XY.

#### Note

We can draw a line perpendicular to the middle point of a line segment using a set square.

#### Note

We can also bisect a line segment using the measurement on a ruler. We measure the length of the line segment with the ruler and then plot a point in the middle of its length. This is not a geometrical construction, just a measurement.

Helpful notes adding information or clarifying details about the theory sections.

### Apply your knowledge

- 6 Use a ruler and a pair of compasses to bisect the line segments.

(a)



(b)



'Apply your knowledge' sections specifically targeted to cover the learning objectives of each unit assisting students in applying and consolidating their newly acquired knowledge of concepts and processes.

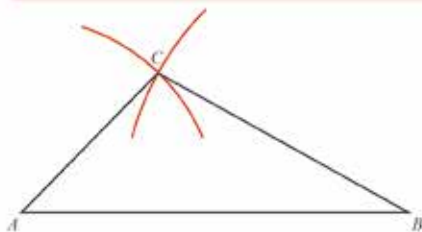


Detailed theory sections including various methods and worked examples for representation of the main mathematical concepts and further understanding of the mathematical methodologies.

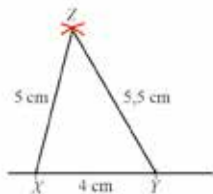
## 9.5 Constructing triangles

We can construct a triangle  $ABC$  with all three sides of known length using a ruler and a pair of compasses following the steps below:

- Step 1:** We use a ruler to draw a line segment with length equal to one side of the triangle ( $\overline{AB}$ ) and name its end points.
- Step 2:** We measure the length of the second side ( $\overline{AC}$ ) on a ruler with the compasses. We place the sharp point of the compasses on end point  $A$  and turn the drawing hand around to draw an arc.
- Step 3:** We repeat the procedure of step 2 using the length of the third side ( $\overline{BC}$ ), placing the sharp point of the compasses on end point  $B$ .
- Step 4:** The two arcs we drew before meet at a point that is vertex  $C$  of the triangle. We plot point  $C$  and use a ruler to join the vertices  $A$ ,  $B$  and  $C$  of the triangle.



Construct triangle  $XYZ$  with  $\overline{XY} = 4$  cm,  $\overline{XZ} = 5$  cm and  $\overline{YZ} = 5,5$  cm.



- Step 1:** Use a ruler to draw the line segment  $\overline{XY}$  with length 4 cm.
- Step 2:** With a compass measuring 5 cm, place the sharp point at point  $X$  and make an arc.
- Step 3:** With a compass measuring 5,5 cm, place the sharp point at point  $Y$  and make an arc.
- Step 4:** Label where the two arcs intersect  $Z$ , and use a ruler to join  $\overline{XZ}$  and  $\overline{YZ}$ .

### Using tech in maths...

For further exploration you can visit [geogebra.org](http://geogebra.org) and try constructing triangles with sides of several lengths, using circles.

'Using tech in maths...' sections with websites for further exploration.

### Think deeper

Can you construct a different triangle with the same given measurements?

'Think deeper' sections with questions and problems to trigger students' interest.

'All about maths' sections with historical information related to each topic.

'!' sections helping students to avoid serious mathematical mistakes.

**5 Algebraic expressions**

**5.1 Algebraic expressions**

**All about maths**  
The Persian mathematician and astronomer al-Khwarizmi (c. 800 AD) is considered to be the father of algebra. The word 'algebra' comes from the Arabic al-jabr, which means 'to put together'. The word 'algebra' is also used to describe the study of numbers and their properties. The algebraic expressions are used to represent numbers and their properties. The algebraic expressions are used to represent numbers and their properties. The algebraic expressions are used to represent numbers and their properties.

**Definition**  
An algebraic expression is an expression that contains one or more variables. It is called an algebraic expression because it contains variables. The variables are letters that represent numbers. The variables are letters that represent numbers. The variables are letters that represent numbers.

**Examples**  
1. Simplify the following algebraic expressions:  
(a)  $3x + 5x$  (b)  $2x + 3x + 4x$   
(c)  $5x + 2x + 3x + 4x$  (d)  $3x + 5x + 2x + 4x$   
(e)  $5x + 2x + 3x + 4x + 1x$

**2. Evaluate the following algebraic expressions:**  
(a)  $3x + 5x$  when  $x = 2$  (b)  $2x + 3x + 4x$  when  $x = 3$   
(c)  $5x + 2x + 3x + 4x$  when  $x = 4$  (d)  $3x + 5x + 2x + 4x$  when  $x = 5$   
(e)  $5x + 2x + 3x + 4x + 1x$  when  $x = 6$

**3. Evaluate the following algebraic expressions:**  
(a)  $3x + 5x$  when  $x = 2$  (b)  $2x + 3x + 4x$  when  $x = 3$   
(c)  $5x + 2x + 3x + 4x$  when  $x = 4$  (d)  $3x + 5x + 2x + 4x$  when  $x = 5$   
(e)  $5x + 2x + 3x + 4x + 1x$  when  $x = 6$

**Like terms**  
In the algebraic expression  $3x + 5x + 2x + 4x + 1x$ , the algebraic terms that are equal to  $3x + 5x + 2x + 4x + 1x$  are called like terms.

**Examples**  
1. Simplify the following algebraic expressions:  
(a)  $3x + 5x$  (b)  $2x + 3x + 4x$   
(c)  $5x + 2x + 3x + 4x$  (d)  $3x + 5x + 2x + 4x$   
(e)  $5x + 2x + 3x + 4x + 1x$

**2. Evaluate the following algebraic expressions:**  
(a)  $3x + 5x$  when  $x = 2$  (b)  $2x + 3x + 4x$  when  $x = 3$   
(c)  $5x + 2x + 3x + 4x$  when  $x = 4$  (d)  $3x + 5x + 2x + 4x$  when  $x = 5$   
(e)  $5x + 2x + 3x + 4x + 1x$  when  $x = 6$

'Exercises' section with numerous graded activities where students apply their knowledge in different contexts in order to enable them to develop their problem solving skills.

'Maths as language' section summing up the wording of core mathematical symbols.

**7 Linear equations and Inequalities**

**Exercises**

- Solve the equations:  
(a)  $3x + 5 = 2x + 1$  (b)  $4x + 3 = 2x + 1$  (c)  $5x + 2 = 3x + 1$  (d)  $6x + 4 = 3x + 1$
- Solve the equations:  
(a)  $3x + 5 = 2x + 1$  (b)  $4x + 3 = 2x + 1$  (c)  $5x + 2 = 3x + 1$  (d)  $6x + 4 = 3x + 1$
- Solve the equations:  
(a)  $3x + 5 = 2x + 1$  (b)  $4x + 3 = 2x + 1$  (c)  $5x + 2 = 3x + 1$  (d)  $6x + 4 = 3x + 1$
- Given that  $x = \frac{1}{2}$ , find the value of  $y$ , when  $x = 1$  and  $y = 2$ .
- Three consecutive integers have a sum equal to 63. Form an equation and find the three integers.
- Jack is  $y$  years old. His brother is 3 years older than him. The sum of their ages is 34 years. How old is Jack?
- Solve the linear inequalities:  
(a)  $3x + 5 < 2x + 1$  (b)  $4x + 3 < 2x + 1$  (c)  $5x + 2 < 3x + 1$  (d)  $6x + 4 < 3x + 1$
- Solve the inequality  $2x + 3 < 4x + 5$  and find the greatest possible integer value of  $x$ .
- (a) Solve the inequality  $3x + 5 < 2x + 1$ .  
(b) Find the smallest possible integer value of  $x$  for which  $3x + 5 < 2x + 1$ .
- A rectangle has sides of  $(x + 1)$  cm and  $(2x + 3)$  cm.  
(a) Given that the perimeter is 40 cm, find an inequality and show that  $x > 5$ .  
(b) Find the smallest possible value of  $x$ .  
(c) Find the perimeter of the rectangle for this value of  $x$ .

**More exercises**

- The length of a rectangle is  $(2x + 3)$  cm and the breadth is 7 cm. If its area is 70 cm<sup>2</sup>, find  $x$ .
- The equal sides of an isosceles triangle are each equal to twice the length of its third side. If its third side is  $y$  cm long and its perimeter is 38 cm, find the length of all the sides.
- A pencil costs 4 cents and a ruler costs twice as much. The total cost of 15 pencils and 4 rulers is 140p. Find the cost of a ruler and the cost of a pencil.
- The height of two buildings are  $(3x + 2)$  m and  $(2x + 1)$  m. The difference in height of the two buildings is 7 m. What is the value of  $x$ ?
- $x$  is a positive whole number. When we subtract 4 from it and then multiply the result by 5, the answer is less than 16. Form a linear inequality in terms of  $x$  and find the values of  $x$ .

**Maths as language**

$x = y + 4$	$x$ is 4 more than $y$ .
$y = x + 4$	$y$ is 4 more than $x$ .
$x = y - 4$	$x$ is 4 less than $y$ .
$y = x - 4$	$y$ is 4 less than $x$ .
$x = y$	$x$ is equal to $y$ .
$y = x$	$y$ is equal to $x$ .
$x > y$	$x$ is greater than $y$ .
$y > x$	$y$ is greater than $x$ .
$x < y$	$x$ is less than $y$ .
$y < x$	$y$ is less than $x$ .
$x \geq y$	$x$ is greater than or equal to $y$ .
$y \geq x$	$y$ is greater than or equal to $x$ .
$x \leq y$	$x$ is less than or equal to $y$ .
$y \leq x$	$y$ is less than or equal to $x$ .

**Unit at a glance**

- A linear equation is an equation in which each term can be written as a single variable multiplied by a coefficient. All variables should be raised to the power of 1.  
Linear equations:  $2x + 3 = 5$ ,  $x + 1 = 2$   
Non-linear equations:  $x^2 + 3x + 2 = 0$
- A linear inequality is an equation in which each term can be written as a single variable multiplied by a coefficient. All variables should be raised to the power of 1.  
Linear inequalities:  $2x + 3 < 5$ ,  $x + 1 < 2$   
Non-linear inequalities:  $x^2 + 3x + 2 < 0$
- A fractional equation is an equation where one or more of the coefficients of the unknown variable is a fraction.  
Fractional equations:  $\frac{1}{x} + \frac{2}{x} = 3$ ,  $\frac{1}{x} + \frac{2}{x} = 3$
- Solving a linear equation means finding the value of the unknown quantity. We can check whether our answer is correct or not by replacing the variable in the original equation with the value we found.
- The simultaneous equations that have two or more variables and are linear are called linear equations. We say that  $x$  is the subject of a formula if  $x$  is expressed in terms of other variables. When we find the value of the subject of a formula, we express the output.
- Solving a word problem using an equation, expression requires forming an equation and then solving it. When we solve a word problem, we should always check whether the answer we find is acceptable.
- An inequality is a mathematical expression with two parts separated by an inequality sign. A linear inequality is an inequality in which each term contains only linear expressions.

'More exercises' section with activities specifically designed to challenge students and extend their knowledge and problem-solving skills.

'Unit at a glance' section summarising the core mathematical terms and concepts taught in each unit.

## 6 Coordinates and linear graphs

### Assessment

Read the questions carefully. For each question, 4 options are given. Circle the correct one.

- Which of the points lies on the  $x$ -axis?  
(a) (5, 3) (b) (-3, 0) (c) (0, 3) (d) (5, 3)
- Which of the points lies on the  $y$ -axis?  
(a) (2, 3) (b) (-4, -3) (c) (5, 3) (d) (5, -3)
- Which of the points is in the 2nd quadrant?  
(a) (3, -4) (b) (-5, 4) (c) (-4, -2) (d) (5, 0)
- Three of the vertices of a square are  $(-1, 2)$ ,  $(-4, 5)$ ,  $(2, 2)$ . What are the coordinates of the fourth vertex?  
(a)  $(-2, -5)$  (b)  $(5, 2)$  (c)  $(2, 5)$  (d)  $(3, 3)$
- What is the shape formed by the points  $(-2, 3)$ ,  $(5, 0)$ ,  $(5, 4)$ ,  $(4, 3)$ ,  $(-2, 3)$ ?  
(a) rectangle (b) square (c) trapezium (d) rhombus
- Among the points, which one is the nearest to the origin?  
(a) (5, 4) (b)  $(-5, 0)$  (c) (5, 5) (d) (5, 3)
- Which vector expresses the translation of a shape 3 units left and 9 units up?  
(a)  $\vec{d} = (-3, -9)$  (b)  $\vec{d} = (-3, -9)$  (c)  $\vec{d} = (3, -9)$  (d)  $\vec{d} = (-3, 9)$
- Which vector expresses the translation of a shape 3 unit right and 8 units down?  
(a)  $\vec{d} = (3, -8)$  (b)  $\vec{d} = (8, -3)$  (c)  $\vec{d} = (3, 8)$  (d)  $\vec{d} = (-3, 8)$
- Given the equation of the line,  $y = 3x - 2$ , what is the value of  $y$  when  $x = 1$ ?  
(a) 1 (b) -1 (c) 2 (d) -2
- Given the equation of the line,  $y = 6 + 2x$ , what is the value of  $y$  when  $x = 2$ ?  
(a) 3 (b) 4 (c) 8 (d) 2

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'Assessment' questions at the end of each unit for revision and consolidation of the main mathematical concepts.

## Glossary

cumulative frequency	the sum of the absolute frequencies of all previous classes, when shown in a table.
decimal fraction	a fraction with a power of 10 as its denominator.
decimal place	the position of a digit in the fractional part of a decimal number.
decomposition	the method of analysing a number into parts depending on the place value.
denominator	the lower part of a fraction.
descending	from the greatest to the smallest.
diagonal	a line segment that joins two non-adjacent vertices of a polygon.
digit	the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
direction	(for vectors) the angle that a vector forms with the horizontal axis.
distributive property	for any numbers and variables $a$ , $b$ and $c$ it is true that $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$ .
divisor	the number we divide by.
equivalent fraction	a fraction that has the same value as another, but a different numerator and denominator.
evaluate	to find the arithmetic value of a mathematical expression of a variable.
event	a set of outcomes from the sample space of an experiment.
expand	the opposite of factorise.
exponent	the symbol or number written at the upper right side of a number that shows how many times this number is multiplied by itself. $a^n = a \times a \times \dots \times a$ ( $n$ times).
exterior	outside the shape.
factorise	write an algebraic expression in the form of a product.
factorisation	the process of changing an algebraic expression to an equivalent in the form of a product of two or more factors.
favourable outcome	the result we are interested in getting.
formula	a mathematical equation which is used to calculate something specific and often two or more variables and is always true for the values of the variables.
general statement	a mathematical equation or sentence that is always true.
graph	a diagram that represents the relation between two variables in a coordinate system.
grouped data	data that has been organised into a frequency table in order to be sorted into classes.
identity property	for any number or variable $a$ it is true that $a + 1 = a$ and $a \times 1 = a$ .
image	the new position of an object after translation.

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Glossary with age-appropriate definitions of critical mathematical terms at each level ensuring the gradual development of mathematical vocabulary.

## Review 1-6

- Represent the numbers on the number line.  
(a) odd numbers between 16 and 25  
(b) numbers smaller than 5  
(c) numbers bigger than or equal to 10 and smaller than 20  
(d) 5, 9, 13 and 18
- Complete the sentences.  
(a) The absolute value of  $-47$  is \_\_\_\_\_.  
(b) The graph of quantities that are in inverse proportion is a \_\_\_\_\_.  
(c) 13.9% is equal to the decimal \_\_\_\_\_.  
(d) The method of analysing a number into parts depending on the place value is called \_\_\_\_\_.  
(e) The number 9 to the 7th is called the \_\_\_\_\_.
- Write the numbers in order using  $<$  or  $>$  signs.  
(a) Ascending order: 7, -12, 4, 0, 8  
(b) Descending order:  $\frac{4}{5}$ ,  $\frac{3}{4}$ ,  $\frac{2}{3}$ ,  $\frac{1}{2}$ ,  $\frac{1}{10}$   
(c) Ascending order: 3.13, 5.07, 3.009, 3.12, 3.6  
(d) Descending order:  $\frac{5}{10}$ ,  $\frac{6}{10}$ ,  $\frac{7}{10}$ ,  $\frac{8}{10}$ ,  $\frac{9}{10}$
- Change mixed numbers into improper fractions, proper fractions into decimals and decimals into decimal fractions or vice versa.  
(a)  $5\frac{4}{5} =$  \_\_\_\_\_ (b) 0.006 = \_\_\_\_\_ (c)  $7\frac{3}{10} =$  \_\_\_\_\_  
(d)  $\frac{725}{1000} =$  \_\_\_\_\_ (e)  $\frac{42}{50} =$  \_\_\_\_\_ (f)  $\frac{156}{100} =$  \_\_\_\_\_

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## Review 7-12

- Solve the equations.  
(a)  $8x - 3 = 5 + 2x$  (b)  $3(2x - 5) + 2x = 7(x + 5)$  (c)  $\frac{x}{12} + 10 = 30$
- Given that  $\frac{2B + A}{18} = \frac{C}{5}$ , find the value of  $A$  when  $B = 9$  and  $C = 27$ .
- Solve the linear inequalities.  
(a)  $4x - 15 \geq x - 12$  (b)  $5(x - 2) < 5(-x + 1)$  (c)  $2(\frac{x}{8} + 1) \geq 18$

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Review pages with activities covering the first and the second half of the book.

Supplementary section with theory, worked examples or tips assisting students in completing the activities.

Activities categorised according to the difficulty level into 3 categories.

# Review 7-12

**8** Solve the equations.

(a)  $2x + 8 = 34$

(b)  $\frac{x}{4} = 22$

(c)  $7 + 2x = 93 - 2x$

**9** Solve the inequalities and show their solutions on a number line.

(a)  $8x + 12 < 5x + 2$

(b)  $\frac{1}{2}x - 2 \geq 5$

(c)  $8y - 21 < 8y + 12$

**10** Read the sentences and write Yes or No.

(a) Trapeziums have two pairs of parallel sides. \_\_\_\_\_  
 (b) Squares have perpendicular diagonals. \_\_\_\_\_  
 (c) The angles of an isosceles triangle are equal. \_\_\_\_\_  
 (d) The sum of the angles in a triangle is  $90^\circ$ . \_\_\_\_\_

**11** If  $ED = 18$  cm,  $APCD$  is a square and the area of the triangle is  $54 \text{ cm}^2$ , find the area of the composite shape.

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Two Review sections, in the middle and at the end of the Workbook, designed to provide the students with an opportunity to review and consolidate the main mathematical concepts and processes taught in the series.



Map of the units shows the mathematical content of each unit and enables teachers to monitor the progression of knowledge throughout the units.

Map of the units			
Unit	Theory sections	Learning objectives	Keywords
<b>9</b> Congruent figures and constructions	9.1 Congruent figures 9.2 Congruent triangles 9.3 Tests for congruence 9.4 Bisecting a line segment 9.5 Constructing triangles 9.6 Quadrilateral constructions	<ul style="list-style-type: none"> <li>Recognise congruent figures.</li> <li>Realise that congruent figures may have different orientations but are identical in shape.</li> <li>Use tests of congruency to verify that two triangles are congruent.</li> <li>Practice working with congruent triangles and quadrilaterals.</li> <li>Practice making geometrical constructions with the use of compasses, a ruler and a protractor (draw an arc, a circle, bisect a line segment or an angle, draw a line perpendicular to a straight line, etc.).</li> <li>Solve problems involving congruent triangles and other congruent figures.</li> </ul>	<ul style="list-style-type: none"> <li>congruent</li> <li>corresponding</li> <li>angle</li> <li>side-side-side test (SSS)</li> <li>side-angle-side test (SAS)</li> <li>angle-side-angle test (ASA)</li> </ul>
<b>10</b> Statistics	10.1 Basics of statistics 10.2 Tables and frequency tables 10.3 Relative and cumulative frequencies 10.4 Histograms 10.5 Bar charts 10.6 Pie charts 10.7 Line graphs	<ul style="list-style-type: none"> <li>Recognise the influence of statistics in our daily life.</li> <li>Collect, classify and organise data in tables.</li> <li>Use tables and absolute and cumulative frequency to tables or graphs.</li> <li>Construct, read and interpret pictographs, bar charts, bar graphs and line graphs.</li> <li>Recognise the advantages and disadvantages of the different ways of representing data when using statistics.</li> <li>Comprehend how using the wrong graphs and tables to using them at the wrong way can lead us to interpret the data incorrectly.</li> </ul>	<ul style="list-style-type: none"> <li>represent</li> <li>population</li> <li>representative sample</li> <li>quantitative variables</li> <li>qualitative variables</li> <li>ally mean</li> <li>class</li> <li>frequency</li> <li>relative frequency</li> <li>cumulative frequency</li> <li>pie chart</li> <li>bar chart</li> <li>line graph</li> </ul>
<b>11</b> Data analysis	11.1 Data analysis 11.2 The mean of ungrouped data 11.3 Median 11.4 Mode 11.5 The mean of grouped data	<ul style="list-style-type: none"> <li>Identify the mean, the mode and the median of a set of data.</li> <li>Realise the purposes and uses of the mean, the mode and the median of a set of data.</li> <li>Calculate all the different types of average and gain information about a data set.</li> <li>Realise that when the mean of a data set is not equal to the mode, it is useful to find the median to obtain a better estimation for the data set.</li> <li>Draw conclusions about the properties of a survey through different types of averages.</li> <li>Familiarise ourselves with the concept of probability as a measure of chance.</li> <li>Familiarise ourselves with the random experiment, the outcome, the sample space and the event.</li> <li>Use a formula to find the probability of an event.</li> <li>Use the properties of probability to solve problems.</li> <li>Calculate the probability of a single event.</li> <li>Compare the probability of an event at random with its relative frequency in a digital experiment.</li> <li>Use tables, bar graphs, etc. to represent all possible outcomes of an experiment.</li> </ul>	<ul style="list-style-type: none"> <li>ungrouped data</li> <li>mean</li> <li>mode</li> <li>median</li> <li>discrete</li> <li>grouped data</li> </ul>
<b>12</b> Probability	12.1 Introduction to probability 12.2 Definitions of probability 12.3 Properties of probability 12.4 Tree diagrams	<ul style="list-style-type: none"> <li>Familiarise ourselves with the concept of probability as a measure of chance.</li> <li>Familiarise ourselves with the random experiment, the outcome, the sample space and the event.</li> <li>Use a formula to find the probability of an event.</li> <li>Use the properties of probability to solve problems.</li> <li>Calculate the probability of a single event.</li> <li>Compare the probability of an event at random with its relative frequency in a digital experiment.</li> <li>Use tables, bar graphs, etc. to represent all possible outcomes of an experiment.</li> </ul>	<ul style="list-style-type: none"> <li>probability</li> <li>random</li> <li>random experiment</li> <li>outcome</li> <li>sample space</li> <li>event</li> <li>addition</li> <li>probability of the event</li> <li>random</li> <li>certain event</li> <li>impossible event</li> <li>tree diagram</li> <li>event</li> </ul>

Step-by-step guidelines for the corresponding Student's Book theory section and teaching notes facilitating the teaching of the new concepts and processes.

**9.4 Bisecting a line segment**

Draw SA's attention to the theory section Bisecting a line segment.

Explain to SA that to bisect a line segment using a pair of compasses and a ruler, we draw a perpendicular line that crosses through the middle point of the line segment.

**Notes:**

Point out to SA that we can also draw a line perpendicular to the middle point of a line segment using a set square.

Focus SA's attention on the picture, and explain to SA that we first place the compass at one end of the line segment, point 1, then we adjust the compass to slightly longer than half the line segment and we draw an arc. Next, keeping the compass at the same length, we draw an arc (2) with the compass at point 2, and we join point 1 to point 2, cutting the line segment (3) at (c). The line segments (3) and (4) have the same length. So, we have bisected (1).

**Notes:**

Point out to SA that we can also bisect a line segment using the measurement on a ruler. We measure the length of the line segment with the ruler and then plot a point at the middle of its length. This is not a geometrical construction, just a measurement.

Have SA do the activities in the Apply your knowledge section.

**9.5 Constructing triangles**

Draw SA's attention to the theory section Constructing triangles.

Focus SA's attention on the triangle ABC, and explain to SA that we can construct a triangle ABC with all three sides of known length using a ruler and a pair of compasses.

Explain to SA that to do so, we first use a ruler to draw a line segment with length equal to one side of the triangle (AB) and name its end points, then we measure the length of the second side (AC) on a ruler with the compasses, place the sharp point of the compass on end point A and turn the drawing hand around to draw an arc. We then repeat the procedure using the length of the third side (BC), placing the sharp point of the compass on end point B. The two arcs we have drawn meet at a point that is vertex C of the triangle. Finally, we join point C and use a ruler to join the vertices A, B and C of the triangle.

Answers to each activity of the Student's Book.



## 8 Scatter plot and population analysis

## 8.1 Scatter plots with quantitative variables

## Note

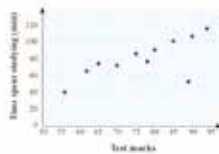
The variables we study in statistics can be sorted into quantitative and qualitative variables. Quantitative variables can have numerical values, such as the age or height of students in a class or the time they spend studying, while qualitative variables cannot be measured numerically but can be placed into categories, such as eye colour, political preferences, etc.

We already know that statistics is a branch of mathematics we use to collect and interpret data. The analysis of data collection helps us make predictions, take decisions, or even come to useful conclusions. There are techniques in statistics through which we can identify whether a variable has a relationship with a different variable, or if they progress independently. Such techniques include the scatter plot graphs. A **scatter plot** is a graphical representation of a set of data points in a coordinate plane. Each point represents the value of two variables, one variable on the horizontal axis and one on the vertical axis. The position of each point corresponds to the relation of the values of the two variables for that point. The distribution of the data on the coordinate plane helps us identify patterns, relationships, and **tendencies** between two variables.

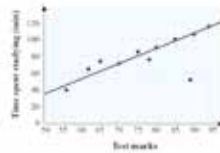
For example, we can study whether there is any relationship between the time somebody spends studying for a maths test and the marks scored in this test. The table shows the marks in the test of 11 students and the time they studied for this test.

Student	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Mark	88	75	78	56	94	85	62	89	79	81	90	80	70	80
Time (min)	90	85	78	40	75	78	65	59	72	102	107	95	88	90

We plot each point in the coordinate plane, where the x-axis represents the test marks and the y-axis represents the time spent studying of each student.



Each point corresponds to a student, and its position is defined by the values of each variable. We notice that the cloud of points shows a relationship between the two variables. As the time spent studying increases, the test marks increase as well. So, there is an increasing tendency of the two variables. **Intuitively**, we can use the pattern of the points to form a line that highlights this tendency.

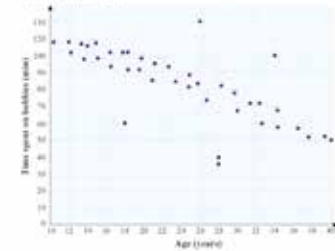


When we can imagine and draw a line that shows the tendency of the two variables, we say that there is a relationship of **correlation** between them. Specifically, when the points in a scatter plot form a clear pattern, line appears to be modelled as a straight line, then the variables are **linearly correlated**. We say that the closer the data are to the line, the stronger relationship the variables have between them.

We also notice that the point (59, 51) is isolated from the basic cloud of data on the plane. This could have happened due to a measurement error or, rarely, an extreme value that does not follow the pattern of the data. This point is called an **outlier**.

## Example

- A survey studied the time 40 people of ages 15–40 spent daily doing their hobbies. The data is represented by a scatter plot.
- Determine if the variables are linearly correlated and if so, draw a line to show the correlation.
  - Find the outliers, if there are any.
  - What could be a conclusion considering the relation between age and the time spent on hobbies?



## 8

## Note

When we draw the line to highlight the linear correlation of a dot data set, we don't take into consideration the outliers. In real-life surveys where we normally work with a large number of variables, we reject the outliers before we use technological tools to verify and visualise the correlation.

## 4 Linear relations and simultaneous equations

## 1 Read the sentences and write Yes or No.

- The gradient of a straight line  $ax + by = k$  is given by  $-\frac{a}{b}$ .
- If  $A(x_1, y_1)$  and  $B(x_2, y_2)$  are two points that lie on a straight line, then its gradient is equal to  $\frac{y_2 - y_1}{x_2 - x_1}$ .
- When the system of two simultaneous equations has no solutions, the graphs of the equations are parallel lines.
- For two lines with equations  $ax + by = k$  and  $ax + by = k$ , if  $\frac{a}{a} \neq \frac{b}{b}$ , then the lines intersect at one point.
- When the gradient of a straight line is undefined, the line is perpendicular to the x-axis.
- When we solve a system of simultaneous equations with two different methods, we expect different solutions from each one.
- A linear relation  $fx + cy = ax + by$  can represent a straight line on a coordinate plane.

2 Transform the equation into the form  $y = mx + c$  and find their gradient.

## Tip

The equation of the form  $ax + by = k$ , where  $a, b, k \in \mathbb{Q}$ , and  $b \neq 0$ , can be expressed as  $y = -\frac{a}{b}x + \frac{k}{b}$ , where its gradient is  $-\frac{a}{b}$ .

- $2x - 3y = 4$
- $2x + 5y = 5$
- $-\frac{1}{3}x - \frac{4}{3}y = \frac{1}{3}$
- $4.5x - 5y = 2.25$

## 4

3 Find the gradient of the equations and transform them into the form  $ax + by = k$ .

- $(a) x = 7x + 7$
- $(b) y = -5x + 6$
- $(c) y = -\frac{3}{5}x + \frac{4}{12}$
- $(d) y = -1.5x + \frac{3}{2}$

## 4 Use the graphical method to solve the simultaneous equations.

## Number of solutions of simultaneous equations

- When the graphs of the linear equations have one point of intersection, the coordinates of the point of intersection give the solution of the simultaneous equations.
- If the lines are parallel, the lines will never intersect, so the simultaneous equations have no solution.
- If the lines are the same, there is an infinite set of common points that satisfy both equations.

## Example

Solve the system of simultaneous equations:

$$\begin{cases} x - y = -1 \\ 2x + y = 4 \end{cases}$$

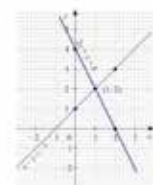
Make a table for each equation.

x	y
0	1
1	2

x	y
0	4
2	0

Plot the points (0, 1) and (2, 2) for the linear equation  $x - y = -1$ . Plot the points (0, 4) and (2, 0) for the linear equation  $2x + y = 4$ . Join the points with straight lines and label the lines on the same system of axes.

The coordinates of the point of intersection (1, 2) is the solution of the simultaneous equations. So, the solution is  $x = 1, y = 2$ .





# Explorers!

## Maths



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